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107/5097 66 DT04 Rec'd PCT/PTO 2 8 SEP 2004

Title: Wireless communication using multi-transmit multi-receive antenna arrays

Description

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5 Field of the invention

This invention relates to wireless communication using multi-transmit multireceive antenna arrays, that is to say where both the transmitting and the receiving station comprise an array of antenna elements. In cases where the antenna elements at a given station may be used both for transmission and for reception, references herein to a 'transmitter', a 'transmit antenna' a 'receiver' or a 'receive antenna' are to be construed as references to the function that they are exercising during that operation.

Background of the invention

Wireless communication systems are assuming ever-increasing importance for the transmission of data, which is to be understood in its largest sense as covering speech or other sounds and images, for example, as well as abstract digital signals.

Currently proposed standards for wireless communication systems include the 3GPP (3rd generation Partnership Project) and 3GPP2 standards, which use Code Division Multiple Access ('CDMA') and Frequency Division Duplex ('FDD') or Time Division Duplex ('TDD'), the HIPERLAN and HIPERLAN2 local area network standards of the European Telecommunications Standards Institute ('ETSI'), which use Time Division Duplex ('TDD') and the International Telecommunications Union ('ITU') IMT-2000 standards. The present invention is applicable to systems of these kinds and other wireless communication systems.

In order to improve the communication capacity of the systems while reducing the sensitivity of the systems to noise and interference and limiting the power of the transmissions, various techniques are used separately or in combination, including space-time diversity, where the same data is transmitted over different transmit and/or receive antenna elements, and frequency spreading.

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such as Orthogonal Frequency Division Multiplex ('OFDM') where the same data is spread over different channels distinguished by their sub-carrier frequency.

At the receiver, the detection of the symbols is performed utilising knowledge of the complex channel attenuation and phase shifts: the Channel State Information ('CSI'). The Channel State Information is obtained at the receiver by measuring the value of pilot signals transmitted together with the data from the transmitter. The knowledge of the channel enables the received signals to be processed jointly according to the Maximum Ratio Combining technique, in which the received signal is multiplied by the Hermitian transpose of the estimated channel transfer matrix.

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Two broad ways of managing the transmit diversity have been categorised as 'closed loop' and 'open loop'. In closed loop signal transmission, information concerning the transmission channels is utilised at the transmitter to improve the communication. For example, the document Tdoc SMG2 UMTS-L1 318/98 presented to the ETSI UMTS Physical Layer Expert Group describes operation of a Transmit Adaptive Array (Tx AA) FDD scheme in which the dedicated channels are transmitted coherently with the same data and code at each transmit antenna, but with antenna-specific amplitude and phase weighting. The receiver uses pilots transmitted on the Common Channels to estimate separately the channels seen from each antenna. The receiver estimates the weights that should be applied at the transmitter to maximise the power received at the receiver, quantises the weights and feeds them back to the transmitter. The transmitter applies the respective quantised weights to the amplitudes and phases of the signals transmitted from each transmit antenna of the array. Alternatively, in TDD systems, the channel state information for weighting the signals applied to the downlink transmit antennas may be derived from the uplink signals, assuming that the channels are equivalent, without transmission of any specific channel or weighting information from the receiver to the transmitter.

Multi-Transmit-Multi-Receive ('MTMR') diversity schemes, where essentially the same signal is transmitted in space-time diversity over the different combinations of transmit and receive antenna elements, can provide significant gains in Signal-to-Noise Ratios ('SNR') and thus operate at low SNRs, enabling an

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increase in spectral efficiency via the use of high order modulations. Alternatively, in multi-stream wireless communication schemes, different signals can be transmitted between the transmit and receive antenna element arrays enabling high spectral efficiency. However, multi-stream schemes of this kind that have been proposed are viable only at high SNRs and require complex receivers (for a N-Transmit and M-Receive antenna configuration, M must be greater than or equal to N) in order to be able to extract the different transmitted signals at the receiver.

An example of an open-loop multi-stream single user scheme is the Bell Labs layered space-time ('BLAST') scheme described in an article by G. J. Foschini entitled "Layered Space-Time Architecture for Wireless Communication in a fading Environment When Using Multiple Antennas," Bell Laboratories Technical Journal, Vol. 1, No. 2, Autumn, 1996, pp. 41-59.

A closed-loop alternative to the above scheme in which channel knowledge is used at the transmitter for multi-stream transmission is described in an article by Mansoor Ahmed, Joseph Pautler and Kamyar Rohani entitled "CDMA Receiver Performance for Multiple-Input Multiple-Output Antenna Systems," Vehicular Technology Conference, Fall, Atlanta City, Oct 2001. A schematic diagram illustrating the principle of this communication system is shown in the accompanying Figure 1.

Such schemes are limited by compromises between diversity gain and spectral efficiency and accordingly the range of operational SNRs is limited unless complexity is increased or high modulation constellations (for example greater than 64 QAM) are used. The present invention offers a substantial improvement in the compromise between diversity gain and spectral efficiency.

Summary of the invention

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The present invention provides a method of, and apparatus for, wireless communication using multi-transmit multi-receive antenna arrays as described in the accompanying claims.

Brief description of the drawings

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Figure 1 is a schematic diagram of a known generic multi-stream single user communication system,

Figure 2 is a schematic diagram of a multi-stream communication system in accordance with one embodiment of the invention, given by way of example,

Figure 3 is a graph illustrating the performance of the system of Figure 2 for different spectral efficiencies,

Figure 4 is a graph illustrating the performance of the system of Figure 2 compared with an open loop system with the same number of transmit antenna elements but different numbers pf receive antenna elements, and

Figure 5 is a graph illustrating the performance of the system of Figure 2 compared with an open loop system with the same numbers of transmit and receive antenna elements.

Detailed description of the preferred embodiments

Figure 1 of the drawings shows a known multi-stream wireless communication system comprising a transmitter station 1 comprising a transmit antenna array 2 of N transmit antenna elements and a receiver station 3 comprising a receive antenna array 4 of M receive antenna elements. In the example illustrated in Figure 1, N=M=2. A plurality of distinct data streams x_1 to x_2 (F=1 two in the example of Figure 1) are transmitted from the transmit antenna array 2 to the receive antenna array 4 and the data streams are weighted by respective complex weighting coefficients $v_{n,f}$ where n is the nth transmit antenna element and f is the fth data stream before being applied to the transmit antenna array. The distinct data streams are separated and estimated at the receiver station in a linear or non-linear receiver 5, to produce detected signals s_1 and s_2 .

In the case shown in Figure 1, with N=M=F=2, the propagation channel can be represented by a matrix $\underline{H} = \begin{bmatrix} \underline{h}_{11} & \underline{h}_{12} \\ \underline{h}_{21} & \underline{h}_{22} \end{bmatrix}$. In the closed-loop system developed by Motorola and described in the article referred to above by Mansoor Ahmed, Joseph Pautler and Kamyar Rohani, channel knowledge is used at the transmitter

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for the multi-stream transmission. This scheme requires the knowledge of the weight matrix, $V = [V1 \ V2]$, applied at the transmit antennas where $V1 = [v_{1,1} \ v_{2,1}]^T$ and $V1 = [v_{2,1} \ v_{2,2}]^T$ are two eigen-vectors of $\underline{H}^H\underline{H}$ (T and H stand for transpose and conjugate transpose respectively). The inputs n_1 and n_2 shown in Figure 1 represent noise added to the signal channels. The noise is assumed in the analysis below to be independent, identically distributed ('i.i.d.') complex-valued Gaussian random values with variance σ^2 (AWGN noise). Finally y_1 and y_2 represent the respective received signals on the two antennas of the receive antenna array 2.

It will be appreciated that the BLAST technique described in the article referred to above by G. J. Foschini is equivalent to setting $v_{1,1}=v_{2,2}=1$ and $v_{1,2}=v_{2,1}=0$, that is to say that each data stream is transmitted only on a single respective transmit antenna element and no channel knowledge is used at the transmitter (open loop).

It will also be appreciated that, in a conventional TxAA closed loop transmit diversity scheme, a single stream is transmitted according to the eigenvector corresponding to the maximum eigenvalue of $\underline{H}^H\underline{H}$, so that $V1=[v_{1,1}\ v_{2,1}]^T$ and V2=0. This is a closed loop single stream single user scheme whereas in the dual-stream TxAA shown in figure 1 both eigenvectors V1 and V2 are used.

Analysis, in the context of High Speed Downlink Shared Channel (HS-DSCH) communication, has arrived at two main conclusions regarding the dual-Stream Tx AA. The first conclusion is that at low SNR (about -5dB), with turbo codes, H-ARQ and water-filling the closed-loop dual-stream scheme can provide up to 50% increase in a verage throughput (Bits/Chip Interval) when compared to the open loop dual-stream scheme. The second conclusion is that for the closed-loop dual-stream scheme the performance (average throughput) of a non-linear receiver is nearly the same as that with a linear receiver, that is to say that the use of channel knowledge at the transmitter eliminates the need for non-linear processing.

However, it has been found that single stream closed loop transmit diversity (Tx AA) provides the best performance at mid and low SNR (-5 to 10 dB) and average throughput of 0.5 to 3 bits/chip-interval. This is very important, given that

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the high SNR scenario (>10 dB) conditions occur with low probability in cellular systems (especially CDMA systems, for example).

The embodiment of the present invention shown in Figure 2 takes advantage of closed loop transmit diversity while increasing the data rate by using multi-streaming. Similar elements in Figure 2 to those of Figure 1 have the same numbering.

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This embodiment of the invention is applicable generally where F data streams are transmitted from respective sub-groups of the transmit antenna elements at least one of which comprises a plurality of the transmit antenna elements. In a preferred embodiment of the invention, each of the sub-groups of transmit antenna elements has the same number N_d of transmit antenna elements. In another embodiment of the present invention, the sub-groups of transmit antenna elements have different numbers of transmit antenna elements, each of the sub-groups comprising at least N_d transmit antenna elements. Preferably, as in this embodiment of the invention, the minimum number N_d of transmit antenna elements in any sub-group is at least two. The use of more than one antenna element in a sub-group improves the diversity of the communication for that data stream, while the use of more than one sub-group improves the spectral efficiency by transmitting different signals via the sub-groups. The choice of the configuration, including the number of transmit antenna elements in each subgroup, and hence of N and N_d is an optimisation problem which can be formulated in the context of a given application as a function of channel conditions and target performance, for example.

Depending on the target performance and functioning SNR, one can choose N_d , and the number of groups in order to provide the needed diversity and spectral efficiency. Moreover, one can also choose to set N_d and the number of groups such that not all N antennas are used, economising on calculation complexity at the receiver. This configuration can be used in the case of good channel quality, thus high SNR and low target performance. In one embodiment of the present invention, the numbers of antennas used in total and in each sub-group and the value of N_d are modified during operation of the system to adapt the choices to the current channel conditions and target performance.

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On the receiver side, this embodiment of the invention is applicable generally to a number M of receive antennas, where M is greater than or equal to (N/N_d) .

For the sake of simplicity, the preferred embodiment of the invention is illustrated in Figure 2 for the case of 2 data-streams, 4 transmit antennas and 2 receive antennas (N=4, $N_d=2$, M=2). The multi-stream wireless communication system shown in Figure 2 comprises a transmitter station 1 comprising a transmit antenna array 2 of two transmit antenna elements and a receiver station 3 comprising a receive antenna array 4 of two receive antenna elements. A linear or non-linear receiver 5 separates, decodes and demodulates the signals received at the receive antenna array 4.

The elements of the transmit antenna array 2 are connected in two subgroups 6 and 7. Two distinct data streams x_1 and x_2 are transmitted respectively from the transmit antenna sub-group 6 and from the transmit antenna sub-group 7 to the receive antenna array 4. The data stream x_1 is weighted by complex weighting coefficients v_1 and v_1 before being applied to the two antenna elements of the sub-group 6 respectively and the data stream x_2 is weighted by complex weighting coefficients v_3 and v_4 before being applied to the two antenna elements of the sub-group 7 respectively. The distinct data streams are separated and estimated at the receiver station in a linear or non-linear receiver 5, to produce detected signals s_1 and s_2 .

In the case shown in Figure 2, with $N=M=N_d=2$, the propagation channel can be represented by two matrices $\begin{bmatrix} \underline{h}_{11} & \underline{h}_{12} \\ \underline{h}_{21} & \underline{h}_{22} \end{bmatrix}$ and $\begin{bmatrix} \underline{h}_{31} & \underline{h}_{41} \\ \underline{h}_{32} & \underline{h}_{42} \end{bmatrix}$, where h_{ij} represents the channel from the i^{th} transmit antenna element to the j^{th} receive antenna element.

The received signal vector can then be represented as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$
 Equation 1

where

$$u_{1} = \begin{bmatrix} \underline{h}_{11} & \underline{h}_{21} \end{bmatrix} \begin{bmatrix} v_{1} \\ \underline{h}_{12} & \underline{h}_{22} \end{bmatrix} \begin{bmatrix} v_{1} \\ v_{2} \end{bmatrix} = \begin{bmatrix} \underline{h}_{11}v_{1} + \underline{h}_{21}v_{2} \\ \underline{h}_{12}v_{1} + \underline{h}_{22}v_{2} \end{bmatrix} , \quad u_{2} = \begin{bmatrix} \underline{h}_{31} & \underline{h}_{41} \end{bmatrix} \begin{bmatrix} v_{3} \\ \underline{h}_{32} & \underline{h}_{42} \end{bmatrix} \begin{bmatrix} \underline{h}_{31}v_{3} + \underline{h}_{41}v_{4} \\ \underline{h}_{32}v_{3} + \underline{h}_{42}v_{4} \end{bmatrix}$$
 Equation 2

and where the data streams are weighted by respective complex weighting coefficients $v_{n,f}$, n being the n^{th} transmit antenna element and f the f^{th} data stream, y_1 and y_2 represent the respective received signals on the two antennas of the receive antenna array 2, and n_1 and n_2 represent noise added to the signal channels, again assumed to be independent, identically distributed ('i.i.d.') complex-valued Gaussian random values with variance σ^2 (AWGN noise).

Re-writing Equation(1) in a vector form, we obtain that:

$$Y = H_{equ} x + N$$
 Equation 3

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$$H_{equ} = \begin{bmatrix} u_1 & u_2 \end{bmatrix} = \begin{bmatrix} \underline{h}_{11}v_1 + \underline{h}_{21}v_2 & \underline{h}_{31}v_3 + \underline{h}_{41}v_4 \\ \underline{h}_{12}v_1 + \underline{h}_{22}v_2 & \underline{h}_{32}v_3 + \underline{h}_{42}v_4 \end{bmatrix}$$
 Equation 4

and the dimension of H_{equ} is 2x2.

The estimated symbols (streams) at the output of a linear minimum mean square error (MMSE) receiver are given by:

$$s = GY = GH_{equ}x + GN$$
 Equation 5

where $G = (H_{equ}{}^H H_{equ} + \sigma^2 I)^{-1} H_{equ}{}^H$ is the transfer function of the MMSE receiver, I is the identity matrix and the superscript H stands for the operation transpose conjugate.

For each stream the coefficients $V_1 = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$ and $V_2 = \begin{bmatrix} v_3 & v_4 \end{bmatrix}^T$ are chosen in order to maximize the received power P under unit norm constraint so that the total transmit power is also normalized. The analytic solutions for V_1 and V_2 , also called the eigenfilter solution (see for example chapters 4.4 and 4.5 of the book "Adaptive filter theory" by S imon Haykin, published by P rentice Hall) are the eigenvectors corresponding to the largest eigenvalues of the matrices $H_1^H H_1$ and $H_2^H H_2$, where

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$$H_1 = \begin{bmatrix} \underline{h}_{11} & \underline{h}_{21} \\ \underline{h}_{12} & \underline{h}_{22} \end{bmatrix} \quad and \quad H_2 = \begin{bmatrix} \underline{h}_{31} & \underline{h}_{41} \\ \underline{h}_{32} & \underline{h}_{42} \end{bmatrix}$$
 Equation 6

Using Equation (6), the two streams can be separated and estimated at the output of the receiver, thus an increase in spectral efficiency of order 2 is obtained. In addition, for coherent combining and diversity gain, the antenna coefficients V_1 and V_2 are chosen to maximize the receiver output power for each stream.

The performance of this embodiment of the present invention, referred to as multi-stream transmit adaptive antenna ('M-TxAA') is shown in Figures 3, 4 and 5 for different values of N, N_d and M and spectral efficiencies, for the case N=4, $N_d=2$ and M=2. The performance is evaluated in terms of un-coded bit error rate ('BER') as a function of the ratio of transmit energy per bit to noise ('Tx Eb/No').

The results obtained with this embodiment of the invention are shown in figure 3 for different spectral efficiencies, given by different coding schemes: binary phase shift key ('BPSK'), quadrature phase shift key ('QPSK'), and quadrature amplitude modulation with 16 and 64 symbols per constellation ('QAM-16' and 'QAM-64').

Figure 4 shows a comparison between the performances of this embodiment of the present invention (M-TxAA) and an open loop system ('OL') with the same number of transmit a ntenna elements (four) and four receive antenna elements instead of this embodiment of the present invention's two receive antenna elements. It will be seen that for the given range of Tx Eb/No [6-20 dB], the performance is significantly improved when M-TxAA is used compared to the multi-stream open loop scheme (BLAST). Furthermore, for a given SNR and uncoded BER, (say 3e⁻² and 20 dB) M-TxAA achieves a bit rate of 12 Bits/Symbol (R=2x6) which is 50% higher than the open loop multi-stream scheme. On the other hand, for a fixed bit rate and a given un-coded BER (e.g. 8 bits/symbol and 3e-2) M-TxAA can operate at a SNR of 16.5 dB which is 3.5 dB less than the open loop multi-stream scheme. Note that for these figures 3 and 4, only M=2 antennas is used at the receiver for M-TxAA, thus resulting in a reduced mobile complexity, whereas the open loop multi-stream needs at least M=4 receive antennas.

Figure 5 shows a comparison between the performances of this embodiment of the present invention (M-TxAA) and an open loop system ('OL') with the same number (four) of transmit antenna elements and of receive antenna elements. It will be seen that, for a given spectral efficiency, e.g. 8bits/symbol, and a given uncoded BER, e.g. 3e⁻², M-TxAA can operate at a SNR of 10.0 dB, which is 10 dB less than the open loop multi-stream scheme. Moreover, at the same uncoded BER of 3e⁻², for a bit rate 50% higher than the open loop (12 bits/symbol rather than 8 bits/symbol), M-TxAA still can operate at an SNR of 14 dB, i.e., 4 dB lower.

The quantisation of the weights $V_1 = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$ and $V_2 = \begin{bmatrix} v_3 & v_4 \end{bmatrix}^T$ can be performed as specified in the current 3GPP Rel'99 Closed loop transmit diversity scheme. The elements v_1 and v_3 can be fixed to a constant power, and v_2 and v_4 are set to relative amplitude and phase (to v_1 and v_3 respectively). Thus only the two coefficients v_2 and v_4 need to be fed back which represents negligible additional overhead.

In the embodiments of the invention described above, the transmit antenna pairs (6) and (7) form part of a single transmitter, that is to say that they are in the same cell/sector. However it is also possible for them to form parts of two different sectors/cells with which the mobile is in simultaneous communication during soft-handover/softer-handover. Thus the mobile would receive, two separate streams from two different cells/sector base-station transmitters.

The embodiments of the invention have been described above with specific reference to the example where there are two transmit antenna sub-groups with two antenna elements in each sub-group and two receive antenna elements. The adaptation of the above equations to the more general case of G sub-groups of transmit antenna elements, the sub-group G_i comprising N_i transmit antenna elements where $N_i \geq N_d$, and M receive antenna elements gives the following equations (indicated for the flat-fading case, the extension to the more general multi-path case being obtained by putting corresponding vectors for the terms of the matrices):

Equation (1) becomes:

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$$\begin{bmatrix} y_1 \\ \vdots \\ y_m \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} u_1 & \cdots & u_g & \cdots & u_G \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_g \\ \vdots \\ x_G \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_g \\ \vdots \\ n_G \end{bmatrix}$$
 Equation 7

The values of u_i become (Equation 2)

$$u_{i} = \begin{bmatrix} \frac{h}{\sum} N_{j} + 1, 1 & \cdots & \frac{h}{\sum} N_{j} + k, 1 & \cdots & \frac{h}{\sum} N_{j}, 1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{h}{\sum} N_{j} + 1, m & \cdots & \frac{h}{\sum} N_{j} + k, m & \cdots & \frac{h}{\sum} N_{j}, m \\ \vdots & \ddots & \vdots & \ddots & \vdots \end{bmatrix} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \vdots \\ \underline{h} \sum_{1 \le j < i} N_{j} + 1, 1 & \cdots & \underline{h} \sum_{1 \le j < i} N_{j} + k, M & \cdots & \underline{h} \sum_{1 \le j \le i} N_{j}, M \end{bmatrix}}_{1 \le j \le i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \vdots \\ \underline{V} \sum N_{j} + 2 \\ \vdots \\ \underline{V} \sum N_{j} \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \vdots \\ \underline{V} \sum N_{j} \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \vdots \\ \underline{V} \sum N_{j} \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \vdots \\ \underline{V} \sum N_{j} \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \vdots \\ \underline{V} \sum N_{j} \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ 1 \le j < i \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j} + 1 \\ \end{bmatrix}}_{1 \le j < i} \underbrace{\begin{bmatrix} \underline{V} \sum N_{j}$$

with i = 1,...,G, note that the sum is for the first index only, that is if we represent $\underline{h}_{index1,index2}$, then index 1 is a sum as expressed above.

Equation 4 becomes:

$$H_{equ} = \begin{bmatrix} u_1 & \cdots & u_g & \cdots & u_G \end{bmatrix} = \begin{bmatrix} H_{equ}(a,b) \\ a = 1 \cdots M \\ b = 1 \cdots G \end{bmatrix}$$
 Equation 9

with

$$H_{equ}(a,b) = \sum_{l=1}^{\nu_b} \underline{h}_{\sum_{i=1}^{N_{f+l},a} \sum_{i=1}^{N_{j+l}} N_{j+l}}$$

Equation 10

The eigenfilter solution for V_i (c.f. Equation 6) is then the eigenvector corresponding to the largest eigenvalue of the matrix $H_i^H H_i$ where:

$$H_{i} = \begin{bmatrix} \underline{h}_{\sum\limits_{1 \leq j < i} N_{j} + 1, 1} & \cdots & \underline{h}_{\sum\limits_{1 \leq j < i} N_{j} + k, 1} & \cdots & \underline{h}_{\sum\limits_{1 \leq j \leq i} N_{j}, 1} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \underline{h}_{\sum\limits_{1 \leq j < i} N_{j} + 1, m} & \cdots & \underline{h}_{\sum\limits_{1 \leq j < i} N_{j} + k, m} & \cdots & \underline{h}_{\sum\limits_{1 \leq j \leq i} N_{j}, m} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \underline{h}_{\sum\limits_{1 \leq j < i} N_{j} + 1, 1} & \cdots & \underline{h}_{\sum\limits_{1 \leq j < i} N_{j} + k, M} & \cdots & \underline{h}_{\sum\limits_{1 \leq j \leq i} N_{j}, M} \end{bmatrix}$$
 Equation 1